Math 130 — Semester 1, 2009
Tutorial exercises — Week 2

There are more tutorial exercises here than your tutor is likely to be able to do during the tutorial class. The idea is for you to practise by doing several examples of each type of exercise. You should prepare these exercises BEFORE your tutorial class, then ask your tutor to do those problems which you found difficult. Work on these exercises with your friends in MATH130, so you can discuss how they are done, and compare answers. If your answers are the same as your friends answers, then it is likely you are correct. In this way you will learn much more than by simply looking up the answers.

1 Calculus

1. Write down the slope, the y-intercept and the x-intercept of the following lines:

(a) $2y = 3x - 5$;

(b) $2y = \frac{1}{2}(x - 5)$;

(c) $x + 2y + 4 = 0$;

(d) $y = 7$.

2. Consider the following pairs of points:

(a) $(-2, -2), (3, 3)$;

(b) $(1, -1), (-1, 1)$;

(c) $(-2, 0), (0, 5)$.

For each pair of points, (a) plot the points and draw the line joining them; (b) without performing any calculations, state whether the line has positive, negative or zero slope; (c) calculate the slope of the line; (d) find the equation of the line passing through both points; (e) find the y-intercept and the x-intercept.

3. State the maximal possible domain (in $\mathbb{R}$), a valid codomain, and the range of each of the following function rules:

(a) $f(x) = x^2 - 5\,,$

(b) $f(x) = \sqrt{5-x}\,,$

(c) $f(x) = \dfrac{1}{x+2} - 2\,.$

2 Algebra

Chen & Duong. Elementary Mathematics. Chapter 1, problems 3, 5 and 6. Please print the problems and bring them to the tutorial.

http://www.maths.mq.edu.au/∼wchen/lnemfolder/em01-ba.pdf

3 Solutions: Calculus

1. To find the slopes, the y-intercepts and the x-intercepts, it helps to rewrite the equations in standard form.

(a) $2y = 3x - 5$ iff $y = \frac{3}{2}\,x - \frac{5}{2}$. The slope is $3/2$, the y-intercept $-5/2$, and the x-intercept $\frac{5/2}{3/2} = \frac{5}{3}$, since $y = 0$ iff $x = 5/3$.

(b) $2y = \frac{1}{2}(x-5) = \frac{1}{2}\,x - \frac{5}{2}$ iff $y = \frac{1}{4}\,x - \frac{5}{4}$. The slope is $1/4$, the y-intercept $-5/4$, and the x-intercept $\frac{5/4}{1/4} = 5$, since $y = 0$ iff $x = 5$.

(c) $x + 2y + 4 = 0$ iff $2y = -x - 4$ iff $y = -\frac{1}{2}\,x - 2$. The slope is $-1/2$, the y-intercept -2, and the x-intercept -4, since $y = 0$ iff $x = -4$.

(d) $y = 7$ iff $y = 0\,x + 7$. The slope is 0, the y-intercept 7, and the x-intercept is not defined, since this line doesn't intersect the x-axis.

2. (a) In case (a), the slope is positive, since the corresponding line is increasing from left to right. Its slope is given by the rise per run, hence by $\frac{3-(-2)}{3-(-2)} = \frac{5}{5} = 1$. Since the slope is 1, the equation of the line has the form $y = x + b$, where we have to find b. The line contains the point $(3, 3)$, and so $3 = 3 + b$, hence $b = 0$. Thus, the equation of the first line is $y = x$. Its y-intercept is 0, and its x-intercept also.

(b) Here, the slope is negative, because the line is tilted downwards going from left to right. The slope is given by $\frac{1-(-1)}{-1-1} = \frac{2}{-2} = -1$. The equation of the line has the form $y = -x + b$, and since the point $(1, -1)$ is on the line, it has to satisfy this condition: $-1 = -1 + b$, and so again $b = 0$. Both its y-intercept and its x-intercept are zero.

(c) This line has a positive slope, since it is tilted upwards. The slope is given by $\frac{5-0}{0-(-2)} = 5/2$. Since the point $(0, 5)$ corresponds with the intersection with the y-axis, the y-intercept equals 5. The point $(-2, 0)$ corresponds with the intersection with the x-axis, and so the x-intercept is -2.

3. The only restriction on the three codomains is that they should contain all possible outputs, that is the ranges. The three prescriptions always result in real numbers, and so the easiest solution for the three codomains is given by $\mathbb{R}$, the set of all real numbers.

(a) $x^2 - 5$ is defined for all real numbers x, and so the maximal possible domain in this case is $\mathbb{R}$. Since $x^2 \geq 0$ for all real numbers x, it follows that $x^2 - 5 \geq -5$ always. On the other hand, each real number larger than or equal to -5 can be generated as an actual output. Therefore, the range consists of all real numbers larger than or equal to -5. Since the graph of this function can be obtained from the graph of the function $g(x) = x^2$ by a vertical translation in the negative direction with five units, this result can also be obtained from inspection of the graph of the squaring function. Algebraically, if $y \geq -5$, then $y + 5 \geq 0$, and so $\sqrt{y+5}$ exists; we then have

$$f(\sqrt{y+5}) = (\sqrt{y+5})^2 - 5 = |y + 5| - 5 = (y + 5) - 5 = y$$

and so the value y can be obtained from the input $\sqrt{y+5}$, provided that $y + 5 \geq 0$, otherwise the input doesn't make sense.

(b) We can only calculate square roots of non-negative numbers, and therefore we have the condition that $5-x \geq 0$, hence $x \leq 5$. The domain in this case consists of all real numbers smaller than or equal to 5, including all negative numbers of course. The square root of a non-negative number returns a non-negative number, and all non-negative numbers occur in the output of this function. Explicitly, if $y \geq 0$, then

$$y = \sqrt{5-x} \quad \text{iff} \quad y^2 = 5 - x \quad \text{iff} \quad x = 5 - y^2$$

Indeed, if $x = 5 - y^2$, then $f(5 - y^2) = \sqrt{5 - (5 - y^2)} = \sqrt{y^2} = |y| = y$, since $y \geq 0$.

(c) The only input where this function rule does not make sense, is when $x = -2$, because in that case $x + 2 = 0$, and we cannot divide by 0; the domain consists of all real numbers except -2. To find the range of this function, it is sufficient to remark that the graph of f can be obtained by translating the graph of the function $g(x) = \frac{1}{x}$ first two units to the left and then two units down. Since the range of g consists of all real numbers except 0 ($1/x$ is never 0), the range of the given function consists of all real numbers except -2. Algebraically, if the number y is in the range of the given function, then there should exist a corresponding input x in the domain of f, and so

$$y = \frac{1}{x+2} - 2 \quad \text{iff} \quad y + 2 = \frac{1}{x+2} \quad \text{iff} \quad \frac{1}{y+2} = x + 2 \quad \text{iff} \quad x = \frac{1}{y+2} - 2$$

The second equivalence only holds when $y \neq -2$, since division by 0 is never allowed, and so -2 is not part of the range! You can easily verify that $f(\dfrac{1}{y+2} - 2) = y$.

4 Algebra

To appear soon (probably)!